Closing Tues: HW 10.1

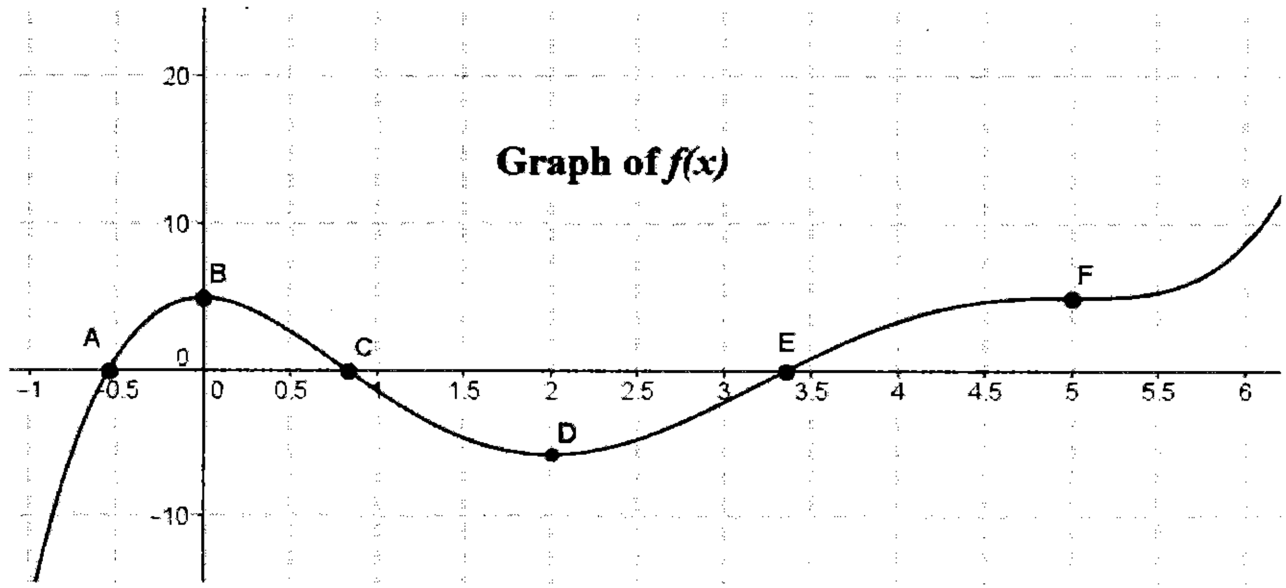
Closing Thurs: HW 10.2

Exam 1 will be returned Tues

10.1 Curve Sketching (relative max/min)

Entry Task: Consider the given $y = f(x)$ graph.

What can you conclude about f'(x) at each of the given points?



$f'>0$ $f'=0$ $f'<0$ $f'=0$ $f'>0$ $f'=0$ $f'>0$

CRITICAL NUMBERS

A $f'(-0.5)$ is positive

B $f'(0) = 0$

C $f'(0.8)$ is negative

D $f'(2) = 0$

E $f'(3.3)$ is positive

F $f'(5) = 0$

f: ↗ ↘ ↗ ↗

f': + 0 − 2 + 5 +

LOCAL MAX

LOCAL MIN

HORIZ. POINT OF INFLECTION

Terminology:

If $f'(x) = 0$ at $x = a$, then we say $x = a$ is a ***critical value*** (or ***critical number***) and $(x, y) = (a, f(a))$ is call a ***critical point***.

We say $y = f(x)$ has a **relative maximum** (or **local maximum**) at $x = a$ if the function changes from increasing to decreasing at $x = a$.

We say $y = f(x)$ has a **relative minimum** (or **local minimum**) at $x = a$ if the function changes from decreasing to increasing at $x = a$.

An ***optimum*** is a max or min.

We say $y = f(x)$ has a **horizontal point of inflection** at $x = a$ if the function does not change direction (stays inc. to inc. or dec. to dec.).

Examples (from Entry Task)

CRITICAL NUMBERS: $x = 0, x = 2, x = 5$

CRITICAL POINTS:

$f(0) = 4 \Rightarrow (x,y) = (0,4)$

$f(2) = -5 \Rightarrow (x,y) = (2,-5)$

$f(5) = 4 \Rightarrow (x,y) = (5,4)$

LOCAL MAXIMUM

occurs at $x = 0$

local max. value $= 4$

LOCAL MINIMUM

occurs at $x = 2$

local min. value $= -5$

OPTIMA

HORIZ. POINT OF INFLECTION

occurs at $x = 5$

h.p.o.i. $= (x,y) = (5,4)$

Once again don't forget the following table which is fundamental to all applications:

$f(x)$	$f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative

NOTES ON SOLVING:

① $\boxed{\text{CLEAR DENOMINATORS}}$

- $\frac{5}{x} = 2 \Rightarrow 5 = 2x$
- $\frac{4}{x^2} - \frac{3}{x} = 1 \Rightarrow 4 - 3x = x^2$
- $\frac{3}{5}x - \frac{1}{10} = 0 \Rightarrow 6x - 1 = 0$

② $\boxed{\text{FACTOR IF POSSIBLE}}$

- $x^3 - 11x^2 = 0 \Rightarrow x^2(x-11) = 0$
- $x^2 - 7x + 12 = 0$

  $(x-3)(x-4) = 0$

  $\Rightarrow x = 3$ or $x = 4$

③ $\boxed{\text{REWRITE EXPONENTS \& simplify}}$

- $(2x-1)^{-10}(x-4) = 0$

  $\Rightarrow \frac{x-4}{(2x-1)^{10}} = 0$

  $\Rightarrow x - 4 = 0$

- $(3x+4)^{-1/2}(2x) - 10(3x+4)^{-1/2} = 0$

  $\Rightarrow (3x+4)^{-1/2}(2x-10) = 0$

  $\Rightarrow \frac{2x-10}{\sqrt{3x+4}} = 0$

  $\Rightarrow 2x - 10 = 0$

  $\Rightarrow x = 5$

Example:

Let $f(x) = x^3 - 6x^2 - 15x + 17$

(a) Find the critical values.

(b) Find the critical points.

(c) Find the intervals over which f(x) is increasing (and over which it is decreasing).

(d) Classify the critical numbers and draw a rough sketch of $y = f(x)$.

$f'(x) = 3x^2 - 12x - 15 \stackrel{?}{=} 0 \quad \div 3$

$x^2 - 4x - 5 \stackrel{?}{=} 0$

$(x-5)(x+1) = 0$

OR, QUADRATIC FORMULA, $x = \frac{4 \pm \sqrt{16 - 4(-5)}}{2}$

$= -1$ OR 5

$x = -1 \Rightarrow f(-1) = (-1)^3 - 6(-1)^2 - 15(-1) + 17$

$= -1 - 6 + 15 + 17 = 25$

$x = 5 \Rightarrow f(5) = (5)^3 - 6(5)^2 - 15(5) + 17$

$= -83$

(a) CRITICAL VALUES: $x = -1$, $x = 5$

(b) CRITICAL POINTS: $(-1, 25)$, $(5, -83)$

(c)

f	↗		↘		↗
f'	+	-1	−	5	+

FOR $x < -1$: Try $x = -2$

$f'(-2) = 3(-2)^2 - 12(-2) - 15$

$= 12 + 24 - 15 > 0$

FOR $-1 < x < 5$: Try $x = 0$

$f'(0) = -15 < 0$

FOR $x > 5$: Try $x = 10$

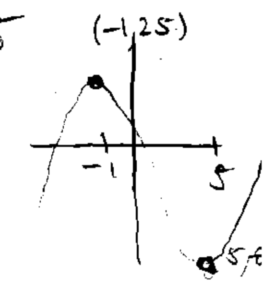
$f'(10) = 3(10)^2 - 12(10) - 15$

$= 300 - 120 - 15 > 0$

(d) CONCLUSIONS:

A <u>local max</u> occurs at $x = -1$.

A <u>local min</u> occurs at $x = 5$



Example:

Let $f(x) = 1 + x^3 - \frac{1}{4}x^4$

(a) Find the critical values.

(b) Find the critical points.

(c) Find the intervals over which f(x) is increasing (and over which it is decreasing).

(d) Classify the critical numbers and draw a rough sketch of $y = f(x)$.

$$f'(x) = 3x^2 - x^3 \stackrel{?}{=} 0$$

$$x^2(3 - x) \stackrel{?}{=} 0$$

$x^2 = 0$ OR $3 - x = 0$

$x = 0$ $\qquad$ $x = 3$

(a) CRITICAL NUMBERS: $x = 0,\ x = 3$

(b) CRITICAL POINTS:

$x = 0 \Rightarrow f(0) = 1 + (0)^3 - \frac{1}{4}(0)^4 = 1$

$(0, 1)$

$x = 3 \Rightarrow f(3) = 1 + (3)^3 - \frac{1}{4}(3)^4 = 28 - \frac{81}{4}$

$= 7.75$

$(3, 7.75)$

(c)

f	$\nearrow$		$\nearrow$		$\searrow$
f'	+	0	+	3	−

FOR $x < 0$: TRY $x = -1$

$f'(-1) = (-1)^2(3 - (-1)) > 0$

FOR $0 < x < 3$: TRY $x = 1$

$f'(1) = (1)^2(3 - 1) > 0$

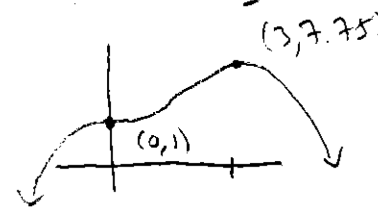
FOR $x > 3$: TRY $x = 10$

$f'(10) = (10)^2(3 - 10) < 0$

(d) CONCLUSIONS

H.P.O.I. $= (0, 1)$

LOCAL MAX OCCURS AT $x = 3$



Example:

Let $f(x) = \sqrt{x^2 + 6x + 100}$

(a) Find the critical values.
(b) Find the critical points.

(c) Find the intervals over which f(x) is increasing (and over which it is decreasing).
(d) Classify the critical numbers and draw a rough sketch of $y = f(x)$.

$$f'(x) = \frac{1}{2}\left(x^2+6x+100\right)^{-1/2} \cdot (2x+6)$$

$$= \frac{2x+6}{2\sqrt{x^2+6x+100}} = \frac{x+3}{\sqrt{x^2+6x+100}} \overset{?}{=} 0$$

$$\Rightarrow \quad x = -3$$

(a) Critical values: $x = -3$

(b) $f(-3) = \sqrt{9-18+100} = \sqrt{91}$
≈ 9.53939

Critical points: $(-3, \sqrt{91})$

(c) f: $\searrow$ ... $\nearrow$

f': $-$ ... -3 ... $+$

For $x < -3$: Try $x = -10$

$$f'(-10) = \frac{-10+3}{\sqrt{(-10)^2+6(-10)+100}} < 0$$

For $x > -3$: Try $x = 0$

$$f'(0) = \frac{0+3}{\sqrt{100}} > 0$$

(d) LOCAL MIN occurs at $x = -3$